VaR Introduction I:
Parametric VaR
Parametric VaR

Summary

- VaR Definition
- VaR Roles
- VaR Pros and Cons
- VaR Approaches
- Parametric VaR
- Parametric VaR Methodology
- Parametric VaR Implementation
- VaR Scaling
- VaR Backtest

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Value at Risk (VaR) Definition

◆ The maximum likely loss on a portfolio for a given probability defined as $x\%$ confidence level over $N$ days

◆ \( \text{Pr}(\text{Loss} > \text{VaR}(x\%)) < 1- x\% \)
VaR Roles

- Risk measurement
- Risk management
- Risk control
- Financial reporting
- Regulatory and economic capital
Parametric VaR

VaR Pros & Cons

◆ Pros
  ◆ Regulatory measurement for market risk
  ◆ Objective assessment
  ◆ Intuition and clear interpretation
  ◆ Consistent and flexible measurement

◆ Cons
  ◆ Doesn’t measure risk beyond the confidence level: tail risk
  ◆ Non sub-additive

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Three VaR Approaches

- Parametric VaR
- Historical VaR
- Monte Carlo VaR

The presentation focuses on parametric VaR.

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Parametric VaR

- **Assumption**
  Asset returns follow normal distribution

- **Pros**
  - Fast and simple calculation
  - Intuitive

- **Cons**
  - Poor accuracy for non-linear products
  - Second order approximation
  - Hard to incorporate stress test

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Assuming an asset return/value change follows normal distribution, the quantile of 99% confidence level is $2.326\sigma$ where $\sigma$ is standard derivation.

If absolute return $X_1 - X_0$ is normally distributed, the 99% worse change of $X$ is $X_1 - X_0 = 2.326\sigma$.

The VaR is given by $\text{VaR} = \frac{\partial F}{\partial X} \Delta X = \frac{\partial F}{\partial X} \times 2.326 \times \sigma$ where $\frac{\partial F}{\partial X}$ is the delta.

Similarly for a relative return $\frac{X_1 - X_0}{X_0}$, the VaR can be expressed as $\text{VaR} = \frac{\partial F}{\partial X} \Delta X = \frac{\partial F}{\partial X} \left( \frac{X_1 - X_0}{X_0} \right) = \frac{\partial F}{\partial X} \times X_0 \times 2.326 \sigma$.
Parametric VaR Implementation

- For each asset/instrument/risk factor, calibrate volatility $\sigma_i$ based on daily return
- For each risk factor pair, calibrate correlation $\rho_{ij}$
- Calculate the variance of a portfolio value change

$$V_p^2 = [\Delta(P_1)\sigma_1 \ldots \Delta(P_n)\sigma_n] \begin{bmatrix} \rho_{11} & \cdots & \rho_{1n} \\ \vdots & \ddots & \vdots \\ \rho_{n1} & \cdots & \rho_{nn} \end{bmatrix} \begin{bmatrix} \Delta(P_1)\sigma_1 \\ \vdots \\ \Delta(P_n)\sigma_n \end{bmatrix}$$

- The portfolio VaR is $2.326 \sqrt{V_p^2}$
Parametric VaR

VaR Scaling

- Normally firms compute 1-day 99% VaR
- Regulators require 10-day 99% VaR
- Under IID assumption, 10-day VaR = \( \sqrt{10} \times VaR_{1-day} \)
Parametric VaR

VaR Backtest

- The only way to verify a VaR system is to backtest
- At a certain day, compute hypothetic P&L. If (hypothetic P&L > VaR) ➔ breach, otherwise, ok
- Hypothetic P&L is computed by holding valuation date and portfolio unchanged
- In one year period,
  - If number of breaches is 0-4, the VaR system is in Green zone
  - If number of breaches is 5-9, the VaR system is in Yellow zone
  - If number of breaches is 10 or more, the VaR system is in Red zone

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Thanks!

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