Financial Sensitivity
Sensitivity

Summary

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Financial Sensitivity Definition

- Financial sensitivity is the measure of the value reaction of a financial instrument to changes in underlying factors.
- The value of a financial instrument is impacted by many factors, such as interest rate, stock price, implied volatility, time, etc.
- Financial sensitivities are also called Greeks, such as Delta, Gamma, Vega and Theta.
- Financial sensitivities are risk measures that are more important than fair values.
- They are vital for risk management: isolating risk, hedging risk, explaining profit and loss, etc.
Delta Definition

- Delta is a first-order Greek that measures the value change of a financial instrument with respect to changes in the underlying asset price.

- Interest rate Delta:
  \[ IrDelta = \frac{\partial V}{\partial r} = \frac{V(r + 0.0001) - V(r)}{0.0001} \]
  where \( V(r) \) is the instrument value and \( r \) is the underlying interest rate.

- PV01, or dollar duration, is analogous to interest rate Delta but has the change value of a one-dollar annuity given by
  \[ PV01 = V(r + 0.0001) - V(r) \]
Credit Delta applicable to fixed income and credit product is given by

\[ \text{CreditDelta} = \frac{\partial V}{\partial c} = \frac{V(c + 0.0001) - V(c)}{0.0001} \]

where \( c \) is the underlying credit spread.

CR01 is analogous to credit Delta but has the change value of a one-dollar annuity given by

\[ PV01 = V(r + 0.0001) - V(r) \]

Equity/FX/Commodity Delta

\[ \text{Delta} = \frac{\partial V}{\partial S} = \frac{V(1.01S) - V(S)}{0.01 * S} \]

where \( S \) is the underlying equity price or FX rate or commodity price.
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Vega Definition

- Vega is a first-order Greek that measures the value change of a financial instrument with respect to changes in the underlying implied volatility.

\[
Vega = \frac{\partial V}{\partial \sigma} = \frac{V(\sigma + \Delta \sigma) - V(\sigma)}{\Delta \sigma}
\]

where \( \sigma \) is the implied volatility.

- Only non-linear products, such as options, have Vegas.

Gamma Definition

- Gamma is a second order Greek that measures the value change of a financial instrument with respect to changes in the underlying price.

\[
Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{V(S + 0.5 \cdot \Delta S) + V(S - 0.5 \cdot \Delta S) - 2V(S)}{\Delta S^2}
\]
Theta Definition

- Theta is a first order Greek that measures the value change of a financial instrument with respect to time.

\[ \text{Theta} = \frac{\partial V}{\partial t} = \frac{V(t + \Delta t) - V(t)}{\Delta t} \]

Curvature Definition

- Curvature is a new risk measure for options introduced by Basel FRTB.
- It is a risk measure that captures the incremental risk not captured by the delta risk of price changes in the value of an option.

\[ \text{Curvature} = \min\{V(S + \Delta W) - V(S) - \Delta W \ast \text{Delta}, V(S - \Delta W) - V(S) - \Delta W \ast \text{Delta}\} \]

where \(\Delta W\) is the risk weight.
Option Sensitivity Pattern

- Sensitivity behaviors are critical for managing risk.
- **Gamma**
  - Gamma behavior in relation to time to maturity shown below.
  - Gamma has a greater effect on shorter dated options.
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Option Sensitivity Pattern (Cont)

- Gamma behavior in relation to moneyness shown below.
- Gamma has the greatest impact on at-the-money options.
Option Sensitivity Pattern (Cont)

- **Vega**
  - Vega behavior in relation to time to maturity shown below.
  - Vega has a greater effect on longer dated options.
Vega behavior in relation to moneyness shown below.

Vega has the greatest impact on at-the-money options.
Option Sensitivity Pattern (Cont)

- **Theta or time decay**
  - Theta is normally negative except some deeply in-the-money deals.
  - Theta behavior in relation to time to maturity shown below.
  - Theta has a greater effect on shorter dated options.
Option Sensitivity Pattern (Cont)

- Theta behavior in relation to moneyness shown below.
- Theta has the biggest impact on at-the-money options.
The objective of hedging is to have a lower price volatility that eliminates both downside risk (loss) and upside profit.

Hedging is a double-edged sword.

The profit of a broker or an investment bank comes from spread rather than market movement. Thus it is better to hedge all risks.

Delta is normally hedged.

Vega can be hedged by using options.

Gamma is hardly hedged in real world.
Sensitivity Profit & Loss (P&L)

- Hypothetic P&L is the P&L that is purely driven by market movement.
- Hypothetic P&L is calculated by revaluing a position held at the end of the previous day using the market data at the end of the current day, i.e.,
  \[ Hypothetical P&L = V(t - 1, P_{t-1}, M_t) - V(t - 1, P_{t-1}, M_{t-1}) \]
  where \( t-1 \) is yesterday; \( t \) is today; \( P_{t-1} \) is the position at yesterday; \( M_{t-1} \) is yesterday’s market and \( M_t \) is today’s market.
- Sensitivity P&L is the sum of Delta P&L, Vega P&L and Gamma P&L.
- Unexplained P&L = Hypothetical P&L – Sensitivity P&L.
Sensitivity Profit & Loss (Cont)

- **Delta P&L:**
  \[ \text{DeltaP&L} = \text{Delta} \times (S_t - S_{t-1}) \]
  where \( S_t \) is today’s underlying price and \( S_{t-1} \) is yesterday’s underlying price.

- **Vega P&L:**
  \[ \text{VegaP&L} = \text{Vega} \times (\sigma_t - \sigma_{t-1}) \]
  where \( \sigma_t \) is today’s implied volatility and \( \sigma_{t-1} \) is yesterday’s implied volatility.

- **Gamma P&L:**
  \[ \text{GammaP&L} = 0.5 \times \text{Gamma} \times (S_t - S_{t-1})^2 \]
Backbone Adjustment

- Backbone adjustment is an advanced topic in sensitivity P&L.
- It can be best explained mathematically.
- Assume the value of an option is a function of the underlying price $S$ and implied volatility $\sigma$, i.e., $V = F(S, \sigma)$.
- If the implied volatility is a function of the ATM volatility and strike (sticky strike assumption), i.e., $\sigma = \sigma_A + f(K)$, the first order approximation of the option value is

$$\Delta V = \frac{\partial F}{\partial S} dS + \frac{\partial F}{\partial \sigma_A} d\sigma_A = \text{Delta P&L} + \text{Vega P&L}$$

where $\text{Delta P&L} = \frac{\partial F}{\partial S} dS$ and $\text{Vega P&L} = \frac{\partial F}{\partial \sigma_A} d\sigma_A$
If the implied volatility is a function of the ATM volatility and moneyness K/S (sticky moneyness or stricky Delta assumption), i.e., $\sigma = \sigma_A + f(S, K)$, the first order approximation of the option value is

$$\Delta V = \frac{\partial F}{\partial S} dS + \frac{\partial F}{\partial \sigma} d\sigma_A + \frac{\partial F}{\partial S} \frac{\partial \sigma}{\partial S} dS = DeltaP&L + VegaP&L$$

where $DeltaP&L = (\frac{\partial F}{\partial S} + \frac{\partial F}{\partial \sigma} \frac{\partial \sigma}{\partial S})dS$ and $VegaP&L = \frac{\partial F}{\partial \sigma} d\sigma_A$

Under sticky moneyness/Delta assumption, the DeltaP&L above has one more item, i.e., $\frac{\partial F}{\partial \sigma} \frac{\partial \sigma}{\partial S} dS$ that is the backbone adjustment.
Thanks!

You can find more details at https://finpricing.com/lib/IrCurveIntroduction.html